

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Third semester B.Tech degree examinations (S) September 2020

Course Code: MA201**Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS**

Max. Marks: 100

Duration: 3 Hours

PART A*Answer any two full questions, each carries 15 marks*

Marks

- 1 a) Find out and give reason whether
- $f(z)$
- is continuous at
- $z = 0$
- (7)

$$f(z) = \begin{cases} \frac{\operatorname{Re} z}{1 - |z|}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

- b) Determine a so that $u = e^{-\pi x} \cos ay$ is harmonic and then find the harmonic conjugate. (8)
- 2 a) Determine the region of the w -plane into which the triangle formed by $x = 1, y = 1$ and $x + y = 1$ is mapped under the transformation $w = z^2$ (7)
- b) Check whether $f(z) = e^z$ is analytic everywhere. (8)
- 3 a) Find the image of $-\frac{1}{2} \leq x \leq \frac{1}{2}, -\pi < y < \pi$ under $w = e^z$ (7)
- b) Find the linear fractional transformation that maps $0, 1, 2$ onto $1, \frac{1}{2}, \frac{1}{3}$ (8)

PART B*Answer any two full questions, each carries 15 marks*

- 4 a) Evaluate $\int_0^{4+2i} \bar{z} dz$ along the curve given by $z = t^2 + it$ (7)
- b) Evaluate $\int_C \frac{2z-1}{z^2-z} dz$ along the curve $C: |z| = 3$ using Cauchy's integral formula. (8)
- 5 a) Find the Laurent's series expansion of $f(z) = \frac{1}{z^2+3z+2}$ in the region $1 < |z| < 2$ (7)
- b) Find all singularities and the corresponding residues (i) $\frac{8}{1+z^2}$ (ii) $\tan z$ (8)
- 6 a) Evaluate $\int_C \frac{e^z}{\cos \pi z} dz$ where c is the unit circle $|z|=1$ using Residue Theorem. (7)
- b) Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos \theta}$ (8)

PART C*Answer any two full questions, each carries 20 marks*

- 7 a) Solve by Gauss elimination (8)
- $$5x - 6y + 4z = 15, 7x + 4y - 3z = 19, 2x + y + 6z = 46$$

- b) Find the rank of $\begin{bmatrix} 6 & 0 & -2 & 0 \\ 0 & -1 & -1 & 5 \\ 2 & -1 & -1 & 0 \end{bmatrix}$ (6)
- c) Let $V = \{(v_1, v_2, v_3) \in R^3 : 3v_2 + v_3 = 2\}$. Is V a vector space under the usual operations in R^3 ? (6)
- 8 a) Find the eigen values and eigen vectors of $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (10)
- b) Is the matrix $A = \frac{1}{9} \begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$ orthogonal? (5)
- c) Check whether $\{(2,0,0,7), (2,0,0,8), (2,0,0,9), (2,0,1,0)\}$ are linearly independent in R^4 (5)
- 9 a) Diagonalize $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ (8)
- b) Transform to principal axis and find what kind of conic section is given by the quadratic form $4x^2 + 12xy + 13y^2 = 16$ (12)
